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Variable Structure Control of Unsteady Aeroelastic System with Partial State Information

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I. Introduction

AEROELASTIC systems exhibit a variety of phenomena including instability, limit-cycle oscillation (LCO), and even chaotic vibration. An excellent survey paper by Mukhopadhyay[1] provides a historical perspective on analysis and control of aeroelastic responses. For uncontrolled (open-loop) aeroelastic systems, several useful studies related to LCOs, bifurcation, flutter instability, etc., have been made.[2–5] Also, synthesis of control systems for controlling unstable responses of aeroelastic models using feedback linearization, adaptive, passivity, and optimal control theory has been performed.[6–9] This Note focuses on the design of a variable structure control (VSC) system[10] for aeroelastic models with parameter uncertainties. Although reduced-order approximations based on quasi-steady aerodynamics are often used for simplicity, here similarly to the use in Ref. 7, the unsteady aerodynamics are included to obtain a more realistic representation. The chosen model describes the plunge and pitch motion of a wing and has a single control surface. For the derivation of the VSC law, the aeroelastic model is treated as the interconnection of two subsystems S_w and S_f. The subsystem S_w describes the pitch, plunge, and control surface motion, and the subsystem S_f is associated with the unsteady aerodynamics. Interestingly, the S_f subsystem is shown to be input-to-state stable (ISS). Because only the states associated with S_w are measured, and the state variables of S_f cannot be measured, the ISS property of the subsystem S_f is exploited to generate a dominating signal using a first-order dynamic system for the synthesis of the VSC law. This way, the estimation of the state variables of S_f, which is a difficult problem for uncertain nonlinear systems, is avoided. Interestingly, the structure of the controller is independent of the dimension of the subsystem S_f. This is important because unsteady dynamics are modeled with an approximation to Theodorsen's theory,[11] yielding models for S_f of different orders. In the closed-loop system, the pitch angle trajectory control is accomplished and the state vectors of S_w and S_f asymptotically converge to the origin in the state space.

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II. Aeroelastic Model and Control Problem

The prototypical aeroelastic wing section is shown in Fig. 1. The governing equations of motion of the aeroelastic system are given by[7,8]

$$\begin{bmatrix} m_t & m_w x_\alpha b \\ m_w x_\alpha b & I_\alpha \end{bmatrix}\begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} c_h \dot{h} \\ c_\alpha \dot{\alpha} \end{bmatrix} + \begin{bmatrix} h k_h(h) \\ \alpha k_\alpha(\alpha) \end{bmatrix} = \begin{bmatrix} -L(t) \\ M(t) \end{bmatrix} \tag{1}$$

where α is the pitch angle, b is the semichord of the wing, h is the plunge displacement, m_w and m_t are the mass of the wing and total mass, I_α is the moment of inertia, and x_α is the nondimensionalized distance of the center of mass from the elastic axis. The lift $L(t)$ and moment $M(t)$ represent the unsteady aerodynamics that are functions of position, velocity, acceleration, and time history of the vortex wave. The lift and moment are acting at the elastic axis of the wing. Here, c_α and c_h are the pitch and plunge damping coefficients, and $k_h(h)$ and $k_\alpha(\alpha)$ are the nonlinear functions associated with the plunge and pitch springs. For purposes of illustration, the functions $k_\alpha(\alpha)$ and $k_h(h)$ are considered as polynomial nonlinearities of fourth and second degree, respectively. These are given by $\alpha k_\alpha(\alpha) = \alpha(k_{\alpha_0} + k_{\alpha_1}\alpha + k_{\alpha_2}\alpha^2 + k_{\alpha_3}\alpha^3 + k_{\alpha_4}\alpha^4)$ and $h k_h(h) = h(k_{h_0} + k_{h_1}h^2)$.

Theodorsen[11] derived the expressions for lift and moment, assuming harmonic motion of the airfoil, of the form (see Ref. 7)

$$\begin{aligned} -L(t) = {} & -\rho b^2 s_p(u\pi\dot{\alpha} + \pi\ddot{h} - \pi b a\ddot{\alpha} - uT_4\dot{\beta} - T_1 b\ddot{\beta}) \\ & - 2\pi\rho u b s_p C(k)\big[u\alpha + \dot{h} + b\big(\tfrac{1}{2} - a\big)\dot{\alpha} \\ & + (1/\pi)T_{10}u\beta + b(1/2\pi)T_{11}\dot{\beta}\big] \end{aligned} \tag{2}$$

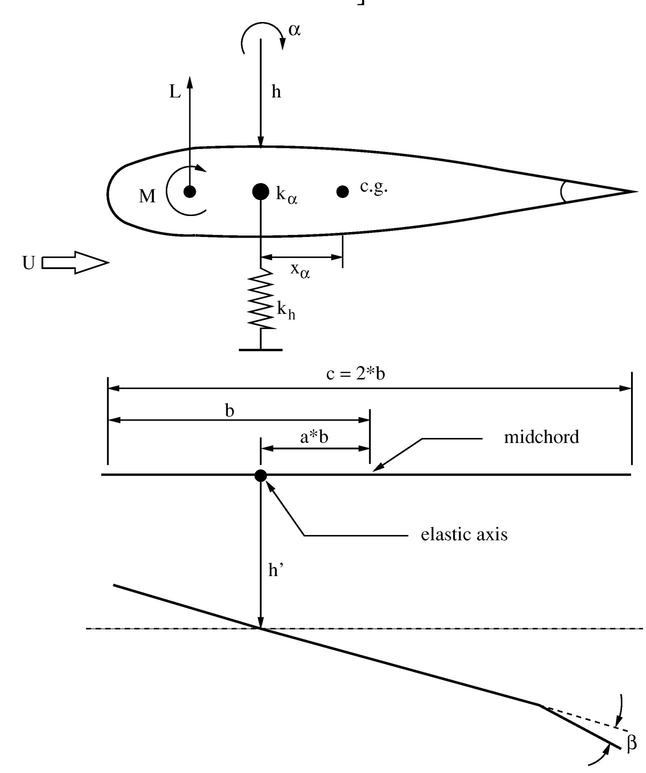

Fig. 1 Aeroelastic model.

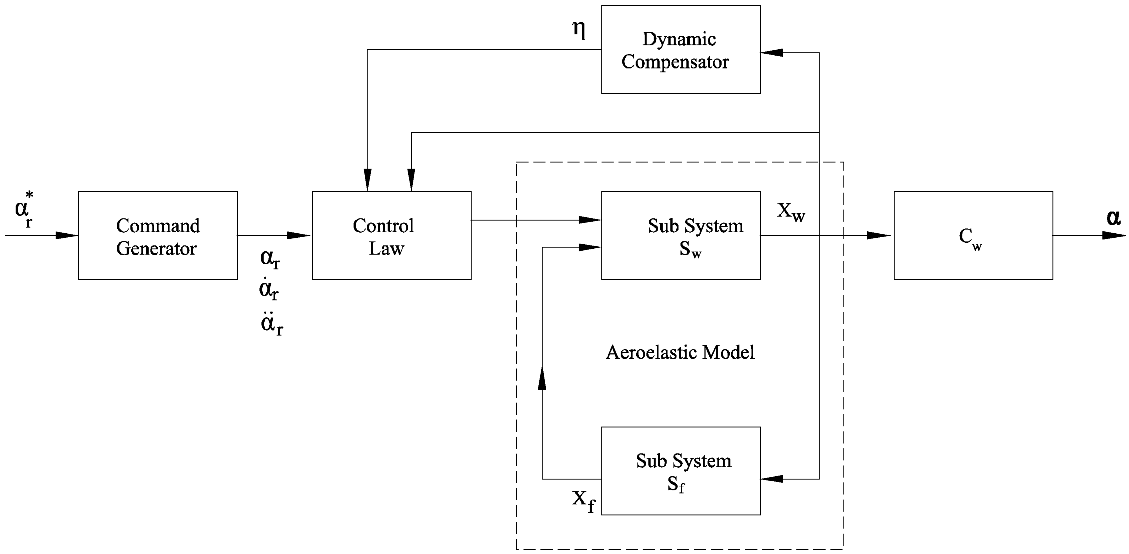

Fig. 2 Block diagram of the aeroelastic model including the variable structure controller.

$$M(t) = -\rho b^2 s_p\{\pi(\tfrac{1}{2}-a)ub\dot{\alpha} + \pi b^2(\tfrac{1}{8}+a^2)\ddot{\alpha} + (T_4+T_{10})u^2\beta + [T_1 - T_8 - (c-a)T_4 + \tfrac{1}{2}T_{11}]ub\dot{\beta} - [T_7 + (c-a)T_1]b^2\ddot{\beta} - a\pi b\ddot{h}\} + 2\rho u b^2 \pi s_p(\tfrac{1}{2}+a)C(k) \times [u\alpha + \dot{h} + b(\tfrac{1}{2}-a)\dot{\alpha} + (1/\pi)T_{10}u\beta + b(1/2\pi)T_{11}\dot{\beta}] \quad (3)$$

where T_i are described by Theodorsen, a is the nondimensionalized distance from the midchord to the elastic axis, and u is the freestream velocity. Jones developed an approximation to Theodorsen's function $C(k)$ given by (see Ref. 7)

$$C(s) \doteq 0.5 + \frac{a_1 s + a_0}{s^2 + p_1 s + p_0} \quad (4)$$

where s is the Laplace variable and $a_1 = 0.1080075u/b$, $a_0 = 0.006825u^2/b^2$ $p_1 = 0.3455u/b$, and $p_0 = 0.01365u^2/b^2$. The control surface dynamics are described by[7]

$$\ddot{\beta} + b_{c1}\dot{\beta} + b_{c0}\beta = b_{c0}\beta_c \quad (5)$$

where $b_{c1} = 50$, $b_{c0} = 2500$, and β_c is the control input to the aeroelastic model.

Theodorsen's function $C(s)$ can be treated as a second-order transfer function of a filter with input

$$v_f(t) = [u\alpha + \dot{h} + b(0.5 - \boldsymbol{a})\dot{\alpha} + (1/\pi)T_{10}u\beta + b(1/2\pi)T_{11}\dot{\beta}] \doteq \boldsymbol{a}_v^T \boldsymbol{x}_w \quad (6)$$

where the vector $\boldsymbol{a}_v \in R^6$ and the partial state vector is $\boldsymbol{x}_w = (h, \alpha, \beta, \dot{h}, \dot{\alpha}, \dot{\beta})^T \in R^6$. The output of the filter is denoted as $y_f(t)$, which is related to the input $v_f(t)$ as $\hat{y}_f(s) = C(s)\hat{v}_f(s)$, where $\hat{y}_f(s)$ and $\hat{v}_f(s)$ represent Laplace transforms of $y_f(t)$ and $v_f(t)$, respectively. The transfer function $C(s)$ of the filter has a minimal realization of dimension two of the form

$$\dot{\boldsymbol{x}}_f = \begin{bmatrix} 0 & 1 \\ -p_0 & -p_1 \end{bmatrix}\boldsymbol{x}_f + \begin{bmatrix} 0 \\ \boldsymbol{a}_v^T \end{bmatrix}\boldsymbol{x}_w \stackrel{\triangle}{=} E_f\boldsymbol{x}_f + l_f\boldsymbol{x}_w \quad (7)$$

where $x_f = (x_{f1}, x_{f2})^T$ and its output is given by

$$y_f = 0.5\boldsymbol{a}_v^T x_w + a_0\boldsymbol{x}_{f1} + a_1 x_{f2} \quad (8)$$

When y_f from Eq. (8) is substituted in Eqs. (2) and (3) and Eqs. (1–5) are used, a state variable representation for the $\boldsymbol{x}_w$ subsystem S_w is obtained, which is given by

$$\dot{\boldsymbol{x}}_w = \begin{bmatrix} O_{3\times 3} & I_{3\times 3} \\ & A_1 \end{bmatrix}\boldsymbol{x}_w + \begin{bmatrix} O_{3\times 2} \\ A_2 \end{bmatrix}\boldsymbol{x}_f + \begin{bmatrix} O_{3\times 1} \\ B_0 \end{bmatrix}\beta_c + \begin{bmatrix} 0_{3\times 5} \\ N_0 \end{bmatrix}\phi(h,\alpha) \doteq A_w\boldsymbol{x}_w + A_f\boldsymbol{x}_f + B\beta_c + N\phi(h,\alpha) \quad (9)$$

where O and I denote null and identity matrices of indicated dimensions; A_i, A_w, A_f, B_0, B, N_0, and N are appropriate constant matrices; and the nonlinear function $\phi(h,\alpha)$ is given by $\phi(h,\alpha) = [h^3\ \alpha^2\ \alpha^3\ \alpha^4\ \alpha^5]^T$. The x_f subsystem S_f [Eqs. (7) and (8)] and x_w subsystem S_w [Eq. (9)] together represent the complete dynamics of the unsteady aeroelastic system. Figure 2 shows a block diagram representation in which the S_f subsystem appears in the feedback path. It is assumed here that the matrices A_1, A_2, B_0, N_0, and a_v and the parameters $p_0 > 0$ and $p_1 > 0$ are unknown. Moreover, only the vector signal $\boldsymbol{x}_w$ is assumed to be measurable, and the state vector $\boldsymbol{x}_f$ of S_f is not available for feedback.

Define the controlled output variable

$$\alpha = [0, 1, 0_{p\times 4}]\boldsymbol{x}_w \stackrel{\triangle}{=} C_w\boldsymbol{x}_w \quad (10)$$

Suppose that $\alpha_r(t)$ is a given smooth reference pitch angle trajectory converging to zero. We are interested in designing a variable structure control law such that in the closed-loop system, the pitch angle follows the reference trajectory $\alpha_r(t)$ and the state vector $(\boldsymbol{x}_w^T, \boldsymbol{x}_f^T)^T$ converges to the origin as well.

III. VSC

In this section, a VSC law[10] for the pitch angle trajectory tracking is derived. For the purpose of design, consider a stable sliding manifold

$$S = \dot{\tilde{\alpha}} + \lambda_1\tilde{\alpha} + \lambda_0 x_s \quad (11)$$

where $\dot{x}_s = \tilde{\alpha}$, $\tilde{\alpha} = \alpha - \alpha_r$ is the tracking error and λ_1 and λ_0 are positive design parameters. In the sliding phase, $S(t) \equiv 0$, and, as such, differentiating Eq. (11) gives

$$\ddot{\tilde{\alpha}} + \lambda_1\dot{\tilde{\alpha}} + \lambda_0\tilde{\alpha} = 0 \quad (12)$$

which implies that $(\tilde{\alpha}, \dot{\tilde{\alpha}})$ tends to zero as $t \to \infty$. Now it remains to derive a control law that makes the sliding manifold attractive.

Differentiating α successively along the trajectory of Eq. (9) gives

$$\dot{\alpha} = C_w A_w \boldsymbol{x}_w$$

$$\ddot{\alpha} = C_w A_w(A_w\boldsymbol{x}_w + A_f\boldsymbol{x}_f + B\beta_c + N\phi) \doteq d\Big[\big(\boldsymbol{a}_w^* + \Delta\boldsymbol{a}_w\big)^T\boldsymbol{x}_w + \big(\boldsymbol{a}_f^* + \Delta\boldsymbol{a}_f\big)^T\boldsymbol{x}_f + \beta_c + \big(\boldsymbol{n}_d^* + \Delta\boldsymbol{n}_d\big)^T\phi\Big] \quad (13)$$

where $d = C_w A_w B$, $a_w = \boldsymbol{a}_w^* + \Delta a_w = (C_w A_w^2)^T d^{-1}$, $\boldsymbol{a}_f = \boldsymbol{a}_f^* + \Delta\boldsymbol{a}_f = (C_w A_w A_f)^T d^{-1}$, and $\boldsymbol{n}_d = (\boldsymbol{n}_d^* + \Delta\boldsymbol{n}_d) = (C_w A_w N)^T d^{-1}$. Here, starred vectors $\boldsymbol{a}_w^*$, $\boldsymbol{a}_f^*$, and $\boldsymbol{n}_d^*$ denote nominal values and $\Delta\boldsymbol{a}_w$, $\Delta\boldsymbol{a}_f$, and $\Delta\boldsymbol{n}_d$ denote bounded uncertainties arising from uncertainties in the model parameters.

Now the following assumption is made.

Assumption: The scalar parameter d is unknown, but its sign is known.

We note that the sign of d can be determined by computing it for the set of model parameters around the nominal values. By the use of Eq. (13), the derivative of S is given by

$$\dot{S} = d\left[\boldsymbol{a}_w^T\boldsymbol{x}_w + \boldsymbol{a}_f^T\boldsymbol{x}_f + \beta_c + \boldsymbol{n}_d^T\phi\right] + \lambda_1\dot{\tilde{\alpha}} + \lambda_0\tilde{\alpha} - \ddot{\alpha}_r \tag{14}$$

which is a function of the state subvector $\boldsymbol{x}_f$ associated with the unsteady aerodynamics. For stability, the control law must attenuate the effect of $\boldsymbol{x}_f$ on the error dynamics. However, $\boldsymbol{x}_f$ is not measurable and cannot be used for feedback. Here, instead of obtaining an estimate of $\boldsymbol{x}_f$, the stability property (to be discussed later) of the subsystem S_f is exploited to construct a signal for feedback.

First, we introduce the definition of ISS from Sontag.[12]

Definition (ISS): The system $\dot{\boldsymbol{q}} = \boldsymbol{g}(\boldsymbol{q}, \boldsymbol{v})$, where $\boldsymbol{g}$ is locally Lipschitz in $\boldsymbol{q} \in R^n$ and the input $\boldsymbol{v} \in R^m$, is said to be ISS if for any $\boldsymbol{q}(0)$, and, for any continuous and bounded $\boldsymbol{v}(t)$ on $[0, \infty)$, the solution exists for all $t \geq 0$ and satisfies $\|\boldsymbol{q}(t)\| \leq \mu[\|\boldsymbol{q}(t_0)\|, t - t_0] + \gamma[\sup\|\boldsymbol{v}(\tau)\|, t_0 \leq \tau \leq t]$ for all t_0 and t such that $0 \leq t_0 \leq t$, where $\mu(s, p)$ and $\gamma(s)$ are strictly increasing functions of $s \in R_+$ with $\mu(0, p) = 0$, $\gamma(0) = 0$, whereas μ is a decreasing function of p with $\lim_{p\to\infty}\mu(s, p) = 0, \forall s \in R_+$. (Here, $\|\ \|$ denotes the Euclidean norm of a vector.)

A. ISS Subsystem S_f and Dynamic Compensator

Indeed, the subsystem S_f [Eq. (7)] is ISS with respect to $\boldsymbol{x}_w$ treated as disturbance input. This can be verified as follows. First we note that E_f is a Hurwitz matrix for each value of the uncertain parameters $p_i > 0$, $i = 1, 2$. Therefore, there exists a positive-definite symmetric matrix P_f (denoted as $P_f > 0$) that satisfies the Lyapunov equation

$$E_f^T P_f + P_f E_f = -I_{2\times 2} \tag{15}$$

Of course, P_f is a function of p_i, that is, $P_f = P_f(p_0, p_1)$. We assume that $p = (p_0, p_1) \in \Omega_f$, a closed and bounded set. Then

$$\lambda_m\|\boldsymbol{x}_f\|^2 \leq \lambda_{\min}(P_f)\|\boldsymbol{x}_f\|^2 \leq \boldsymbol{x}_f^T P_f \boldsymbol{x}_f \leq \lambda_{\max}(P_f)\|\boldsymbol{x}_f\|^2 \leq \lambda_M\|\boldsymbol{x}_f\|^2 \tag{16}$$

where $\lambda_{\min}[\max]$ denotes the minimum [maximum] eigenvalue of P_f, $\lambda_m = \inf_{p\in\Omega_f}\{\lambda_{\min}[P_f(p_0, p_1)]\}$ and $\lambda_M = \sup_{p\in\Omega_f}\{\lambda_{\max}[P_f(p_0, p_1)]\}$.

Now, to verify the ISS property of the subsystem S_f, consider a quadratic Lyapunov function

$$V_f = \boldsymbol{x}_f^T P_f \boldsymbol{x}_f \tag{17}$$

Differentiating V_f along the solution of Eq. (7) gives

$$\dot{V}_f = \boldsymbol{x}_f^T\left(E_f^T P_f + P_f E_f\right)\boldsymbol{x}_f + 2\boldsymbol{x}_f^T P_f l_f \boldsymbol{x}_w \tag{18}$$

By the use of Young's inequality, which states that for all $(x, y) \in R^2$

$$xy \leq kx^2 + (1/4k)y^2$$

Eq. (18) gives (choosing $k = k_f/2$)

$$\left|\boldsymbol{x}_f^T P l_f \boldsymbol{x}_w\right| \leq \|\boldsymbol{x}_f\| \cdot \|P_f l_f\| \cdot \|\boldsymbol{x}_w\| \leq (k_f/2)\|\boldsymbol{x}_f\|^2 + (1/2k_f)\|\boldsymbol{x}_w\|^2 \cdot \|P l_f\|^2 \tag{19}$$

where k_f is a positive real number. Using Eq. (19) in Eq. (18) and choosing $0 < k_f < 1$, one obtains

$$\dot{V}_f \leq -(1 - k_f)\|\boldsymbol{x}_f\|^2 + (1/k_f)\|\boldsymbol{x}_w\|^2\|P_f l_f\|^2 \leq -c_1 V_f + \gamma_1\|\boldsymbol{x}_w\|^2 \tag{20}$$

where $c_1 \leq (1 - k_f)\lambda_M^{-1} \leq \{(1 - k_f)/\lambda_{\max}[P_f(p)]\}$ and $\gamma_1 \geq k_f^{-1}|P_f l_f|^2$.

Solving Eq. (20), one finds that

$$V_f(t) \leq V_f(0)e^{-ct} + (\gamma_1/c_1)\Big[\sup_{\tau\in[0,t)}\|\boldsymbol{x}_w(\tau)\|\Big]^2 \tag{21}$$

The use of Eq. (16) and the inequality $q_1^2 + q_2^2 \leq (q_1 + q_2)^2$ for any two positive real numbers q_1 and q_2, Eq. (21) gives

$$\|\boldsymbol{x}_f(t)\| \leq \left(\lambda_M\lambda_m^{-1}\right)^{\frac{1}{2}}\|\boldsymbol{x}_f(0)\| + \left(\lambda_m^{-1}\gamma_1 c_1^{-1}\right)^{\frac{1}{2}}\sup_{\tau\in[0,t]}\{\|\boldsymbol{x}_w(\tau)\|\} \tag{22}$$

According to the Definition, it follows that the S_f subsystem is ISS with respect to $\boldsymbol{x}_w$ as input.

The response of S_w depends on the signal $\boldsymbol{x}_f$, which is not measured. In Ref. 13, an approach is suggested that is applicable for the stabilization of systems with partial state information without an observer design for state estimation. We adopt this approach here to design the VSC law that avoids feedback of the signal $\boldsymbol{x}_f$. In view of Eq. (20), it is possible to construct a dominating signal $\eta(t)$ that is the solution of a first-order dynamic system satisfying

$$\dot{\eta} = -c_1\eta + \gamma_1\|\boldsymbol{x}_w\|^2 \tag{23}$$

Then it follows that $\eta(t) \geq V_f(t)$, provided that $\eta(0) \geq \lambda_M\|\boldsymbol{x}_f(0)\|^2 \geq V_f(0)$. For such a choice of initial condition $\eta(0)$, one has

$$\eta(t) \geq V_f(t) \geq \lambda_m\|\boldsymbol{x}_f(t)\|^2, \qquad t \geq 0 \tag{24}$$

According to Eq. (24), one observes that $\eta(t)$ is a signal that dominates $\|\boldsymbol{x}_f(t)\|^2$, $t \geq 0$. In the following derivation, instead of $\boldsymbol{x}_f$, the signal $\eta(t)$ will be used for feedback.

B. VSC Law

Now based on the Lyapunov method, the VSC law design will be completed. Consider a Lyapunov function

$$V = S^2/2|d| \tag{25}$$

Differentiating V along the solution of Eq. (15) gives

$$\dot{V} = \mathrm{sgn}(d)\cdot S\left(\boldsymbol{a}_w^T\boldsymbol{x}_w + \boldsymbol{a}_f^T\boldsymbol{x}_f + \beta_c + \boldsymbol{n}_d^T\phi\right) + (\lambda_1\dot{\tilde{\alpha}} + \lambda_0\tilde{\alpha} - \ddot{\alpha}_r)S|d|^{-1} \tag{26}$$

Using Young's inequality and Eq. (24), one can establish the following inequalities:

$$\left|\mathrm{sgn}(d)S\boldsymbol{a}_f^T\boldsymbol{x}_f\right| \leq |S|\cdot\|\boldsymbol{a}_f\|\cdot\|\boldsymbol{x}_f\| \leq |S|\cdot\|\boldsymbol{a}_f\|\left(\eta\lambda_m^{-1}\right)^{\frac{1}{2}} \leq g_1|S|\eta + \left(\|\boldsymbol{a}_f\|^2\big/4g_1\right)\lambda_m^{-1}|S|$$

$$(\lambda_1\dot{\tilde{\alpha}} + \lambda_0\tilde{\alpha} - \ddot{\alpha}_r)S|d_1|^{-1} \leq g_2(\lambda_1\dot{\tilde{\alpha}} + \lambda_0\tilde{\alpha} - \ddot{\alpha}_r)^2|S| + \left(d_1^{-2}\big/4g_2\right)|S|$$

$$\left|\mathrm{sgn}(d)S\Delta\boldsymbol{a}_w^T\boldsymbol{x}_w\right| \leq g_3|S|\cdot\|\boldsymbol{x}_w\|^2 + \left(|\Delta\boldsymbol{a}_w|^2\big/4g_3\right)|S|$$

$$\left|\mathrm{sgn}(d)S\Delta\boldsymbol{n}_d^T\phi\right| \leq g_4|S|\cdot\|\phi\|^2 + \left(\|\Delta\boldsymbol{n}_d\|^2\big/4g_4\right)|S| \tag{27}$$

where g_i, $i = 1, \ldots, 4$, are positive real numbers.

Substituting Eq. (27) and choosing the control law for eliminating all of the unknown functions in Eq. (26), one obtains

$$\beta_c = -\boldsymbol{a}_w^{*T}\boldsymbol{x}_w - \boldsymbol{n}_d^{*T}\phi - G_1\,\mathrm{sgn}(Sd) - G_2\,\mathrm{sgn}(d)S - [g_1\eta + g_2(\lambda_1\dot{\alpha} + \lambda_0\tilde{\alpha} - \ddot{\alpha}_r)^2 + g_3\|\boldsymbol{x}_w\|^2 + g_4\|\phi\|^2](\mathrm{sgn}(Sd)) \tag{28}$$

In view of Eq. (27), substitution of the control law Eq. (28) in Eq. (26) gives

$$\dot{V} \leq -G_1|S| - G_2S^2 + (|S|/4)\Big[\|\boldsymbol{a}_f\|^2(g_1\lambda_m)^{-1} + \left(g_2d_1^2\right)^{-1} + g_3^{-1}\|\Delta\boldsymbol{a}_w\|^2 + g_4^{-1}|\Delta\boldsymbol{n}_d|^2\Big] \leq -G_1|S| - G_2|S^2| + \mu^*|S| \tag{29}$$

where $\mu^* \geq \{\|\boldsymbol{a}_f\|^2(g_1\lambda_m)^{-1} + (g_2 d_1^2)^{-1} + g_3^{-1}\|\Delta\boldsymbol{a}_w\|^2 + g_4^{-1}\|\Delta\boldsymbol{n}_d\|^2\}/4$.

To make $\dot{V}$ negative, one sets the gain G_1 to

$$G_1 = \mu^* + G_0 \tag{30}$$

where $G_0 > 0$. Using Eq. (30) in Eq. (29) gives

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{S^2}{2}\right) \leq |d|\left[-G_0|S| - G_2 S^2\right] \tag{31}$$

According to Eq. (31), the trajectory starting from any initial condition reaches the surface $S = 0$ in a finite time. Subsequently, the trajectory slides along $S = 0$, which, according to Eq. (12), implies that $(\dot{\tilde{\alpha}}, \tilde{\alpha}) \to 0$ as $t \to \infty$. Thus, pitch angle trajectory control is accomplished.

The complete closed-loop system is shown in Fig. 2. The VSC law includes a first-order dynamic compensator [Eq. (23)] in the controller feedback path. Following the control law derivation of this section, it is apparent that the structure of the VSC law is independent of the order of the S_f subsystem, and the dominating signal η generated by only a first-order system is sufficient for control. We note that the system has relative degree two because the second derivative of α depends explicitly on the control input and, as such, has zero dynamics of dimension six. For stability in the closed-loop system, the zero dynamics must be stable. Note that the VSC design can be applied to other models of larger orders; however, it requires a high gain feedback for nullifying the effect of uncertainties. Of course, the VSC law is simpler than the adaptive schemes.

IV. Simulation Results

In this section, the simulation results are presented. The model parameters are taken from Refs. 7 and 8 and are listed in the Appendix. First, the open-loop response is obtained for the chosen initial conditions $x_w(0) = [0.01, 10\text{ deg}, 0, 0, 0, 0]^T$, $x_f(0) = [0, 0.1]^T$ for $u = 16$ m/s and $a = -0.5$. Figure 3 shows that after an initial transient, the pitch angle and the plunge displacement trajectories converge to limit cycles. Apparently, the uncontrolled system is unstable, and the wing undergoes periodic oscillations. (Readers may refer to Ref. 5 for additional results related to the existence of LCOs and domain of stability for this model for other values of u and a.)

Now the simulation of the closed-loop system including the VSC law [Eq. (28)], and the dynamic controller [Eq. (23)] is performed. Although the control law has been derived for uncertainties in each model parameter, here simulation is done for uncertainties in the two key parameters u and a. Note that unlike the uncertainties in u and a, variations in the spring parameters do not pose any special difficulty because these perturbations do not affect the key control input parameter $d = C_w A_w B$ used in design. Of course, LCOs cannot exist without these nonlinear springs. For the choice of the nominal values of $u = 25$ m/s and $a = -0.8$, $\boldsymbol{a}_w^*$ and $\boldsymbol{n}_d^*$ are computed and used in the VSC law (28). The feedback gains are selected as $G_1 = 2.1$, $G_2 = 0.02$, $g_1 = 0.0912$, $g_2 = 0.0912$, $g_3 = 0.0912$, and $g_4 = 0.0912$. The parameters of the sliding manifold are set to $\lambda_1 = 6$ and $\lambda_0 = 9$. The parameters used in Eq. (23) for generating the dominating signal η are selected as $c_1 = 0.0536$ and $\gamma_1 = 0.0628$. For simplicity, the reference pitch angle trajectory is assumed to be zero. Note that the feedback gains satisfying the inequalities of the preceding section are only sufficient for stability in the closed-loop system. Therefore, here these controller parameters have been selected after carrying out several simulations and by observing the simulated responses. Because this discontinuous control law [Eq. (28)] can cause control chattering, a smooth approximation of the signum function by a saturation function with a boundary-layer thickness of $\epsilon = 0.1$ is used. To limit the control surface deflection β, simulation is performed by clamping the magnitude of the control input β_c to a maximum value of 30 deg. Note that the VSC law designed for a higher nominal value of the freestream velocity gives improved responses; therefore, here off-nominal lower values of u are considered for simulation.

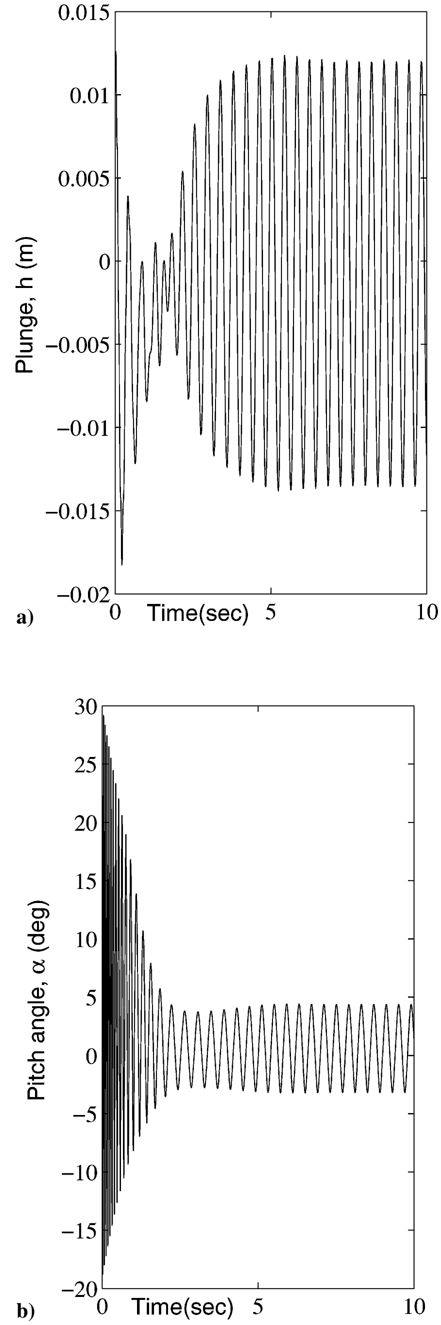

Fig. 3 Open-loop response, $a = -0.5$ and $u = 16$ (m/s): a) plunge displacement (meters) and b) pitch angle (degrees).

The closed-loop system is simulated for the off-nominal values of $u = 16$ m/s and $a = -0.5$. The controller's nominal matrices $(\boldsymbol{a}_w^*, \boldsymbol{n}_d^*)$ in Eq. (13) computed for the nominal values of $u = 25$ m/s and $a = -0.8$ are used for simulation. For the selected off-nominal u and a, it is found that there exists a limit cycle (not shown here for brevity). The linearized system has stable zeros (minimum phase zero dynamics). The closed-loop responses are shown in Fig. 4. The VSC law accomplishes smooth regulation of the pitch angle to zero, but the plunge motion is oscillatory and converges to zero after the initial transient. This oscillatory response of the plunge displacement is attributed to the zero dynamics. Note that the remaining components of the state vector, including the state $\boldsymbol{x}_f$ of the subsystem S_f, also converge to the origin. The response time is of the order of 4 s. The maximum control surface deflection β is less than 25 deg, and the control input β_c saturates over a brief period. Extensive simulation has been performed that shows robustness of the VSC law with respect to uncertainties in u and a.

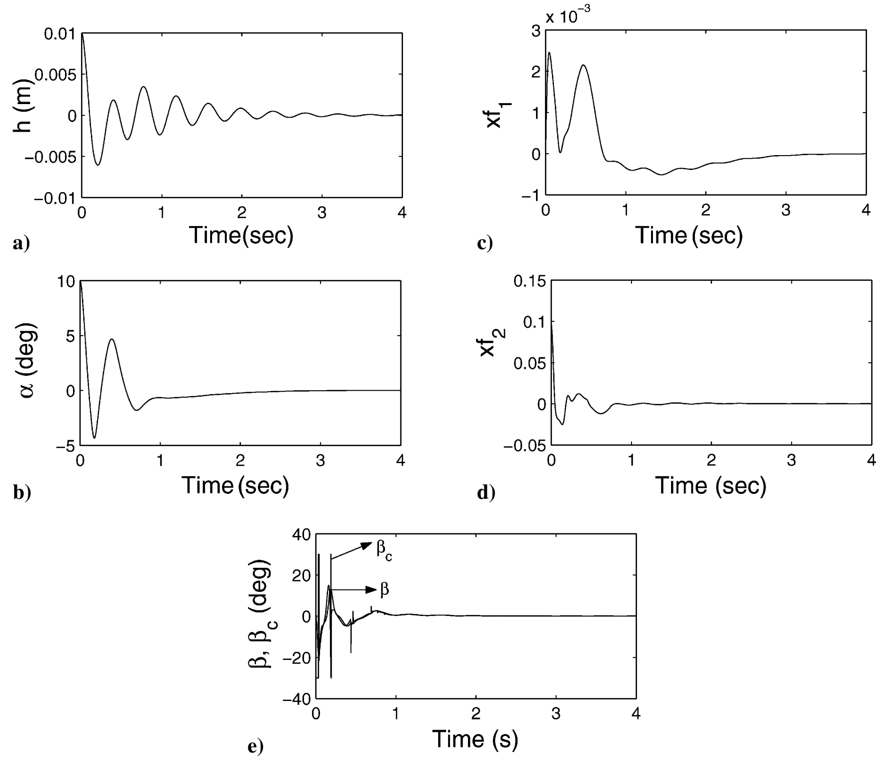

Fig. 4 Closed-loop response, $a = -0.5$ and $u = 16$ (m/s): a) plunge displacement (meters), b) pitch angle (degrees), c) filter state x_{f1}, d) filter state x_{f2}, and e) surface deflection β (degrees) and control input β_c (degrees).

Conclusions

In this Note, control of a prototypical aeroelastic wing section with pitch and plunge structural nonlinearities and unsteady aerodynamics using a single control surface was considered. For the controller synthesis, only the plunge displacement, pitch angle, control surface deflection, and their derivatives were measured. Interestingly, the aeroelastic system can be represented as the interconnection of two subsystems. In this representation, the subsystem associated with the unsteady aerodynamics is ISS. Based on such representation, a variable structure control system was derived. A first-order dynamic system was designed for generating a dominating signal for controller synthesis. Simulation results were presented that showed that flutter suppression can be achieved for uncertainties in the flow velocities and elastic axis locations.

Appendix: System Parameters

The system parameters for simulation have been taken from Refs. 7 and 8:

b = 0.135 m
c_h = 27.43 Ns/m
c_α = 0.036 Ns
I_α = $0.04325 + m_w x_\alpha^2 b^2$ kg · m^2
k_h = $2844.4 + 255.99h^2$ N/m
k_α = $2.82(1 - 22.1\alpha + 1315.5\alpha^2 + 8580\alpha^3 - 17289.7\alpha^4)$ N · m/rad
m_t = 12.387 kg
m_w = 1.662 kg
T_1 = −0.0630
T_4 = −0.4104
T_7 = 0.0128
T_8 = 0.0964
T_{10} = 1.6798
T_{11} = 0.8551
x_α = $[0.0873 - (b + ab)]/b$
ρ = 1.225 kg/m^3

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